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Equality in Soft Erosion and Soft Dilation in Multi Scale Environment

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Abstract: In this paper, equality is discussed in soft-erosion and soft-dilation. In mathematical morphological environment, the property of equality does not exist. But in soft mathematical morphology, the property of equality will exist. It is going to be discussed in this paper in detail.

Keywords: Equality, Mathematical morphology, Mathematical soft morphology, Soft morphology, Erosion, Dilation, Soft erosion, Soft dilation, Primitive morphological operation.

1. INTRODUCTION

It is divided in to three parts. General introduction to Image Processing, introduction to soft morphology and introduction to multi scale mathematical morphology.

1.1. Introduction to Image Processing:

The human beings have the desire of recording incidents, through images. It has started from early cavemen also. Later, so many techniques, to get the images and so many techniques, to process the images are developed. After assembling of computers, image processing was expanded. In 1964 G. Matheron was asked to investigate the relationships between the geometry of porous media and their permeability. At the same time, J. Serra was asked to quantify the petrography of iron ores, in order to predict their milling properties (1, 2). At the same time a centre was developed to study mathematical morphological techniques in Paris school of mines, France. Mathematical morphology can provide solutions to many tasks, where image processing can be applied, such as in remote sensing, optical character recognition, Radar image sequence recognition, medical image processing etc., The image processing algorithms or techniques can be classified in to two categories.1)Linear methods 2)Non Linear methods.

The non linear methods will provide best results, compared to linear methods. The mathematical morphological methods / filters will come under the category of non linear methods/filters. In mathematical morphological operations, Erosion and Dilation are primitive operations (3, 4). But there exist some type of rigidity in mathematical morphological operations. That rigidity is relaxed and the morphological operations are redesigned to overcome some inconveniences, as well as to get some advantages. The primitive operations, Erosion and Dilation, now are called Soft Erosion and Soft Dilation.

1.2. Introduction to soft morphology:

The idea of soft morphological operations is to relax, the standard morphological definition, a little, in such a way that, a degree of Robustness is achieved, While, most of the desirable properties of standard morphological operations are maintained. Soft morphological operators are more tolerant to noise than is provided by erosion and dilation. Soft morphological operators possess many of the characteristics, which are desirable, perform better in noisy environments. (5)In soft morphology, it preserves details, by adjusting its parameters (10). It can be designed in such a way that, it performs well in removal of salt – and – pepper noise as well as Gaussian noise, simultaneously (11).A soft morphological filter can be designed in such a way that, it reduces periodic noise also (12). A filter designed in frequency domain, can function better for smoothening & edge enhancement, according to our requirements. The reason is that by tuning its frequency. But the design involves complex computation. But using soft morphological filters, using very simple computations we can achieve the quality of image processing, to that of filters in frequency domain, which involves complex computations (13).

So, we can conclude that soft morphological filters perform excellent, compared to morphological filters.

1.3 Introduction to multi scale mathematical morphology:

Multi scale morphology has extended its applications to Image Smoothening, Edge Enhancement, Segmentation, Remote Sensing, Radar image analysis, Medical area etc. It is having special applications, like enhancing weak Edges, Decay analysis of wood, critical analysis of (ECG) Cardio imagery (Identification of critical points), Getting results which are helpful for pilots, lunar landing etc.

In the process of understanding the objective world, the appearance of an object does not depend only on the object itself, but also on the scale that the observer used. It seems that appearance under a specific scale does not give sufficient information about the essence of the percept, we want to understand. If we use a different scale, to examine this percept, it will usually have a different appearance. So, this series of images and its changing pattern over scales reflect the nature of the percept. That is why soft erosion and soft dilation are studied in multi scale environment.

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2. DEFINITIONS

In some papers, researchers proposed soft morphology using two sets of structuring elements.

A) The core B) The soft boundary [7, 8, 9].
But, in some papers [5] the scientists/researchers proposed soft morphology, by counting logic. They have done the counting of ones, in the particular sub image, chosen. Then they have applied threshold value, for soft Erosion and soft dilation.
Soft dilation was defined as (5)
$(I \oplus S^{(m)})[x, y] = 1$ If $|I \cap S_{(x,y)}| \geq m$
$= 0$ otherwise.
Here "m" is threshold value where $1 \leq m \leq |S|$. $|S|$ is the cardinality of S.
Soft Erosion may be defined as
$(I \ominus S^{(m)})[x, y] = 0$ If $|\bar{I} \cap S_{(x,y)}| \geq m$
$= 1$ otherwise. $\bar{I}$ = inversion of I; m= threshold $\leq |S|$.
[The exact definition, given in "5", is slightly modified, according to the requirement, but without changing the meaning. Here the main assumption is origin is at central place of the structuring element and structuring element is assumed to be a square grid.]
Here "m" may be taken as 1 to 9, for $3/3$ structuring element or 1 to 25, for $5/5$ structuring element or 1 to 49, for $7/7$ structuring element.

3. DISCUSSION ON SOFT DILATION

3.1 $3/3$ Structuring Elements:

The number of one's (of the sub image) may be ≥ 0 & ≤ 9
If threshold value= 1 then $D_{(1)}$ may be defined as
$(I \oplus S^{(1)})[x, y]=1$ If $|I \cap S_{(x,y)}| \geq 1$
= 0 other wise.
Here, $S^{(1)}$ means, threshold value=1, in the $3/3$ sub image which is chosen, from the image.
If threshold value= 2 then $D_{(2)}$ may be defined as
$(I \oplus S^{(2)})[x ,y]=1$ If $|I \cap S_{(x,y} | \geq 2$
= 0 other wise.
Here, $S^{(2)}$ means, threshold value=2, in the $3/3$ sub image which is chosen, from the image.
If threshold value =3 then $D_{(3)}$ may be defined as
$(I \oplus S^{(3)})[x, y]=1$ If $|I \cap S_{(x,y)}| \geq 3$
= 0 other wise.
Here, $S^{(3)}$ means, threshold value=3, in the $3/3$ sub image which is chosen, from the image.
If threshold = 9 then $D_{(9)}$ may be defined as
$(I \oplus S^{(9)})[x,y]=1$ If $|I \cap S_{(x,y)}| \geq 9$
= 0 other wise.

3.2. $5/5$ Structuring Elements:

For $5/5$ Structuring Element where "m" will run from 1 to 25, the soft dilations are
$D_{(1)}$ to $D_{(25)}$. They may be defined as
If threshold value = 1 then $D_{(1)}$ may be defined as
$(I \oplus S^{(1)})[x, y]=1$ If $|I \cap S_{(x,y)}| \geq 1$
= 0 other wise.
Here, $S^{(1)}$ means, threshold value=1, in the $5/5$ sub image which is chosen, in the image.
If threshold = 2 then $D_{(2)}$ may be defined as
$(I \oplus S^{(2)})[x, y]=1$ If $|I \cap S_{(x,y)}| \geq 2$
= 0 other wise.
Here, $S^{(2)}$ means, threshold value=2, in the $5/5$ sub image which is chosen.
In the same way $D_{(3)}, D_{(4)}, D_{(5)}, D_{(6)}, D_{(7)}, D_{(8)}, D_{(9)}$ may be defined.
If threshold value = 10 then $D_{(10)}$ may be defined as
$(I \oplus S^{(10)})[x, y]=1$ If $|I \cap S_{(x,y)}| \geq 10$
= 0 other wise.
Here, $S^{(10)}$ means, threshold value=10, in the $5/5$ sub image which is chosen.
If threshold value= 11 then $D_{(11)}$ may be defined as
$(I \oplus S^{(11)})[x, y]=1$ If $|I \cap S_{(x,y)}| \geq 11$
= 0 other wise.
Here, $S^{(11)}$ means, threshold value=11 in the $5/5$ sub image, which is chosen.
In the same way $D_{(12)}$, $D_{(13)}$, $D_{(14)}$, ……. $D_{(25)}$ may be defined.

3.3. $7/7$ Structuring Elements:

For $7/7$ Structuring Elements where "m" will run from 1 to 49, the soft dilations are
$D_{(1))}$ to $D_{(49)}$. They may be defined as
If threshold value= 1 then $D_{(1)}$, may be defined as
$(I \oplus S^{(1)})[x, y]=1$ If $|I \cap S_{(x,y)}| \geq 1$
= 0 other wise.
Here, $S^{(1)}$ means, threshold value=1, in the $7/7$ sub image which is chosen.
If threshold value= 2 then $D_{(2)}$, may be defined as
$(I \oplus S^{(2)})[x, y]=1$ If $|I \cap S_{(x,y)}| \geq 2$
= 0 other wise.
Here, $S^{(2)}$ means, threshold value=2, in the $7/7$ sub image which is chosen.
In the same way $D_{(3)}$, $D_{(4)}$, $D_{(5)}$,$D_{(10)}$,………$D_{(25)}$ may be defined.
For threshold value = 26; $D_{(26)}$ may be defined as
$(I \oplus S^{(26)})[x, y]=1$ If $|I \cap S_{(x,y)}| \geq 26$
= 0 otherwise.
Here, $S^{(26)}$ means, threshold value=26, in the $7/7$ sub image which is chosen.
For threshold value = 27; $D_{(27)}$ may be defined as
$(I \oplus S^{(27)})[x, y]=1$ If $|I \cap S_{(x,y)}| \geq 27$
= 0 other wise.
Here, $S^{(27)}$ means, threshold value=27, in the $7/7$ sub

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image which is chosen, in the image.
In the same way $D_{(28)}$, $D_{(29)}$, $D_{(30)}$, $D_{(31)}$,--------- $D_{(49)}$ may be defined.

3.4. $9/9$ Structuring Elements:

For $9/9$ Structuring Elements where "m" will run from 1 to 81; the soft dilations are $D_{(1)}$ to $D_{(81)}$. They may be defined as
If threshold value= 1 then $D_{(1)}$ may be defined as
$(I\oplus S^{(1)})[x, y]=1$ If $|I\cap S_{(x,y)}| =1$
= 0 other wise.
Here, $S^{(1)}$ means, threshold value=1, in the sub image chosen, which having dimension $9/9$.

If threshold value= 2 then $D_{(2)}$ may be defined as
$(I\oplus S^{(1)})[x, y]=1$ If $|I\cap S_{(x,y)}|\geq 2$
= 0 other wise.
Here, $S^{(2)}$ means, threshold value=2, in the sub image chosen, which having dimension $9/9$.

In the same way $D_{(3)}$, $D_{(4)}$,$D_{(5)}$, $D_{(6)}$,-------- $D_{(20)}$,-------- $D_{(79)}$, $D_{(80)}$, $D_{(81)}$ may be defined.

3.5. $11/11$ Structuring Elements:

For $11/11$ Structuring Elements where "m" will run from 1 to 121 , the soft dilations are
$D_{(1)}$, $D_{(2)}$ ……$D_{(121)}$. They may be defined as
If threshold value= 1 then $D_{(1)}$ may be defined as
$(I\oplus S^{(1)})[x, y]=1$ If $|I\cap S_{(x,y)}|\geq 1$
= 0 other wise.
Here, $S^{(1)}$ means, threshold value=1, in the $11/11$ sub image which is chosen, from the image.
If threshold value= 2 then $D_{(2)}$ may be defined as
$(I\oplus S^{(2)})[x, y]=1$ If $|I\cap S_{(x,y)}|\geq 2$
= 0 other wise.
Here, $S^{(2)}$ means, threshold value=2, in the $11/11$ sub image which is chosen, from the image.
In the same way $D_{(3)}$, $D_{(4)}$,………………. $D_{(121)}$ may be defined

3.6. $13/13$ Structuring Elements:

For $13/13$ Structuring Elements where "m" will run from 1 to 169, the soft dilations are $D_{(1)}$, $D_{(2)}$ ……$D_{(169)}$. They may be defined, as defined in the above sections. In the same way, the size of Structuring Elements may be extended to $15/15$, $17/17$, $19/19$ ……. to any dimension, according to our requirement.

4. DISCUSSION ON SOFT EROSION

4.1. $3/3$ Structuring Elements:

In the same way, for Soft Erosion also
If threshold value= 1 then $E_{(1)}$ may be defined as
$(I\ominus S^{(1)})[x , y]=0$ If $|\bar{I}\cap S_{(x,y)}|\geq 1$
= 1 other wise.
Here, $S^{(1)}$ means, threshold value=1, in the $3/3$ sub image which is chosen, from the image.
If threshold value= 2 then $E_{(2)}$ may be defined as
$(I\ominus S^{(2)})[x, y]= 0$ If $|\bar{I}\cap S_{(x,y)}|\geq 2$
= 1 other wise.
Here, $S^{(2)}$ means, threshold value=2, in the $3/3$ sub image which is chosen, from the image.
If threshold value= 9 then $E_{(9)}$ may be defined as
$(I\ominus S^{(9)})[x, y]=0$ If $|\bar{I}\cap S_{(x,y)}|\geq 9$
= 1 other wise.
Here, $S^{(9)}$ means, threshold value=9, in the $3/3$ sub image which is chosen, from the image.

4.2. $5/5$ Structuring Elements:

For $5/5$ Structuring Elements, the thresholds are 1 to 25. The soft erosions may be defined as follows.
If threshold value= 1 then $E_{(1)}$ may be defined as
$(I\ominus S^{(1)})[x, y]=0$ If $|\bar{I}\cap S_{(x,y)}|\geq 1$
= 1 other wise.
Here, $S^{(1)}$ means, threshold value=1, in the sub image chosen, which is having dimension $5/5$ from the image.
If threshold value= 2 then $E_{(2)}$ may be defined as
$(I\ominus S^{(2)})[x, y]=0$ If $|\bar{I}\cap S_{(x,y)}|\geq 2$
= 1 other wise.
Here, $S^{(2)}$ means, threshold value=2, in the sub image chosen, which is having dimension $5/5$ from the image.
If threshold value= 3 then $E_{(3)}$ may be defined as
$(I\ominus S^{(3)})[x, y]=0$ If $|\bar{I}\cap S_{(x,y)}|\geq 3$
= 1 other wise.
Here, $S^{(3)}$ means, threshold value=3, in the sub image chosen, which is having dimension $5/5$ from the image.
In the same way $E_{(4)}$, $E_{(5)}$, $E_{(6)}$, $E_{(7)}$,$E_{(8)}$, $E_{(9)}$.
If threshold value= 10 then $E_{(10)}$ may defined as
$(I\ominus S^{(10)})[x, y]=0$ If $|\bar{I}\cap S_{(x,y)}|\geq 10$
= 1 other wise.
Here, $S^{(10)}$ means, threshold value=10, in the sub image chosen, which is having dimension $5/5$ from the image.
If threshold value= 25 then $E_{(25)}$ may be defined as
$(I\ominus S^{(25)})[x , y]=0$ If $|\bar{I}\cap S_{(x,y)}|\geq 25$
= 1 other wise.
Here, $S^{(25)}$ means, threshold value=25, in the sub image chosen, which is having dimension $5/5$ from the image.

4.3. $7/7$ Structuring Elements

For $7/7$ Structuring Elements, where "m" will be 1 to 49,

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the Soft Erosions are $E_{(1)}, E_{(2)} \ldots\ldots E_{(49)}$ they may be defined as ………
For threshold value= 1 then $E_{(1)}$ may be defined as
$(I \ominus S^{(1)})[x, y]=0$ If $|\bar{I} \cap S_{(x,y)}| \geq 1$
= 1 other wise.
Here, $S^{(1)}$ means, threshold value=1, in the sub image chosen, which is having dimension $7/7$ from the image.
For threshold value= 2 then $E_{(2)}$ may defined as
$(I \ominus S^{(2)})[x, y]=0$ If $|\bar{I} \cap S_{(x,y)}| \geq 2$
= 1 otherwise.
Here, $S^{(2)}$ means, threshold value=2, in the sub image chosen, which is having dimension $7/7$ from the image.
For threshold value= 48 then $E_{(48)}$ may defined as
$(I \ominus S^{(48)})[x, y]=0$ If $|\bar{I} \cap S_{(x,y)}| \geq 48$
= 1 otherwise.
Here, $S^{(48)}$ means, threshold value=48, in the sub image chosen, which is having dimension $7/7$ from the image.
For threshold value= 49 then $E_{(49)}$ may be defined as
$(I \ominus S^{(49)})[x, y]=0$ If $|\bar{I} \cap S_{(x,y)}| \geq 49$
= 1 other wise.
Here, $S^{(49)}$ means, threshold value=49, in the sub image chosen, which is having dimension $7/7$ from the image.

4.4. $9/9$ Structuring Elements

For $9/9$ Structuring Elements, where m=1····· 81, the Soft Erosions are $E_{(1)}$ to $E_{(81)}$. They may be defined as ········
If threshold value= 1 then $E_{(1)}$ may be defined as
$(I \ominus S^{(1)})[x, y]=0$ If $|\bar{I} \cap S_{(x,y)}| \geq 1$
= 1 other wise.
Here, $S^{(1)}$ means, threshold value=1 in the sub image chosen, which is having dimension $9/9$ from the image.
If threshold value= 2 then $E_{(2)}$ may be defined as
$(I \ominus S^{(2)})[x, y]=0$ If $|\bar{I} \cap S_{(x,y)}| \geq 2$
= 1 otherwise.
Here, $S^{(2)}$ means, threshold value=2, in the sub image chosen, which is having dimension $9/9$ from the image.
If threshold value= 3 then $E_{(3)}$ may be defined as
$(I \ominus S^{(3)})[x, y]=0$ If $|\bar{I} \cap S_{(x,y)}| \geq 3$
= 1 other wise.
Here, $S^{(3)}$ means, threshold value=3, in the sub image chosen, which is having dimension $9/9$ from the image.
If threshold value= 79 then $E_{(79)}$ may defined as
$(I \ominus S^{(79)})[x, y]=0$ If $|\bar{I} \cap S_{(x,y)}| \geq 79$
= 1 otherwise.
Here, $S^{(79)}$ means, threshold value=79, in the sub image chosen, which is having dimension $9/9$ from the image.
If threshold value= 80 then $E_{(80)}$ may be defined as
$(I \ominus S^{(80)})[x, y]=0$ If $|\bar{I} \cap S_{(x,y)}| \geq 80$
= 1 otherwise.
Here, $S^{(80)}$ means, threshold value=80, in the sub image chosen, which is having dimension $9/9$ from the image.
If threshold value= 81 then $E_{(81)}$ may be defined as
$(I \ominus S^{(81)})[x, y]=0$ If $|\bar{I} \cap S_{(x,y)}| \geq 81$
= 1 otherwise.
Here, $S^{(81)}$ means, threshold value=81, in the sub image chosen, which is having dimension $9/9$ from the image.

4.5. $11/11$ Structuring Elements

For $11/11$ Structuring Elements, where m=1 to 121, the Soft Erosions are $E_{(1)}, E_{(2)}, E_{(3)}, E_{(4)}, E_{(5)}, \ldots\ldots E_{(121)}$. They may be defined as
If threshold value= 1 then $E_{(1)}$ may be defined as
$(I \ominus S^{(1)})[x, y]=0$ If $|\bar{I} \cap S_{(x,y)}| \geq 1$
= 1 other wise.
Here, $S^{(1)}$ means, threshold value=1 in the sub image chosen, which is having dimension $11/11$ from the image.
If threshold value= 2 then $E_{(2)}$ may be defined as
$(I \ominus S^{(2)})[x, y]=0$ If $|\bar{I} \cap S_{(x,y)}| \geq 1$
= 1 other wise.
Here, $S^{(2)}$ means, threshold value=2, in the sub image chosen, which is having dimension $11/11$ from the image.
If threshold value= 120 then $E_{(120)}$ may be defined as
$(I \ominus S^{(120)})[x, y]=0$ If $|\bar{I} \cap S_{(x,y)}| \geq 120$
= 1 other wise.
Here, $S^{(120)}$ means, threshold value=120, in the sub image chosen, which is having dimension $11/11$ from the image.
If threshold value= 121 then $E_{(121)}$ may be defined as
$(I \ominus S^{(121)})[x, y]=0$ If $|\bar{I} \cap S_{(x,y)}| \geq 121$
= 1 other wise.
Here, $S^{(121)}$ means, threshold value=121, in the sub image chosen, which is having dimension $11/11$ from the image.

4.6. $13/13$ Structuring Elements

For $13/13$ Structuring Elements, where m=1 to 169, the Soft Erosions are $E_{(1)}, E_{(2)}, E_{(3)}, \ldots\ldots E_{(169)}$.
They may be defined, as described in the above sections.

In the same way, the size of Structuring Elements may be extended to $15/15$, $17/17$, $19/19$ ······· to any dimension, according to our requirement.

5. DISCUSSION ON EQUALITY

5.1 Introduction.

By means of the definition of soft erosion and soft dilation the following tables may be constructed. Table 1 will have soft dilation values if the structuring element dimension is $3/3$. So, the thresholds are from 1 to 9.

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Table: 1 Number of 1's of sub image

	0	1	2	3	4	5	6	7	8	9
$D(1)$	0	1	1	1	1	1	1	1	1	1
$D(2)$	0	0	1	1	1	1	1	1	1	1
$D(3)$	0	0	0	1	1	1	1	1	1	1
$D(4)$	0	0	0	0	1	1	1	1	1	1
$D(5)$	0	0	0	0	0	1	1	1	1	1
$D(6)$	0	0	0	0	0	0	1	1	1	1
$D(7)$	0	0	0	0	0	0	0	1	1	1
$D(8)$	0	0	0	0	0	0	0	0	1	1
$D(9)$	0	0	0	0	0	0	0	0	0	1

The following table will give soft erosion values if structuring element size is $^{3}/_{3}$. Here the threshold values are starting from 1 to 9.

Table: 2 Number of 1's of sub image

	0	1	2	3	4	5	6	7	8	9
$E(1)$	0	0	0	0	0	0	0	0	0	1
$E(2)$	0	0	0	0	0	0	0	0	1	1
$E(3)$	0	0	0	0	0	0	0	1	1	1
$E(4)$	0	0	0	0	0	0	1	1	1	1
$E(5)$	0	0	0	0	0	1	1	1	1	1
$E(6)$	0	0	0	0	1	1	1	1	1	1
$E(7)$	0	0	0	1	1	1	1	1	1	1
$E(8)$	0	0	1	1	1	1	1	1	1	1
$E(9)$	0	1	1	1	1	1	1	1	1	1

Table 3 will have soft dilation values if the structuring element dimension is $^{5}/_{5}$. So we get thresholds starting from 1 to 25.

Table.3: Number of 1's of sub image

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
$D(1)$	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$D(2)$	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$D(3)$	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$D(4)$	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$D(5)$	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
….	.	…	…	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
$D(22)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
$D(23)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
$D(24)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
$D(25)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Table 4 will have soft erosion values if the structuring element dimension is $^{5}/_{5}$. So we get thresholds starting from 1 to 25.

Table.4 Number of 1's of sub image

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
$E(1)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
$E(2)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
$E(3)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
$E(4)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
$E(23)$	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$E(24)$	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$E(25)$	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

In the same way we can construct soft erosion and soft dilation values for structuring elements $^{7}/_{7}$, $^{9}/_{9}$, $^{11}/_{11}$, $^{13}/_{13}$ …

5.2. EQUALITY IN SOFT EROSION, SOFT DILATION IN MULTI SCALE ENVIRONMENT:

5.2.1. $^{3}/_{3}$ Structuring Element:

If the Tables 1 and 2 are studied carefully, which provide dilated and eroded values, the conclusion may be got that eroded value at one threshold is equal to dilated value at some other threshold.

For example E (1) is equivalent to D (9). If ,the following, E(1)@D(9) are observed, same output will be obtained , for threshold value 1 in erosion and threshold value 9 in dilation. It is shown below.

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$E(1)$	0	0	0	0	0	0	0	0	0	1

$D(9)$	0	0	0	0	0	0	0	0	0	1

In the same way, E (2) is shown to be equivalent to D (8).

$E(2)$	0	0	0	0	0	0	0	0	1	1

$D(8)$	0	0	0	0	0	0	0	0	1	1

So if we study carefully the following equalities will be observed.

$E(1) = D(9) \quad E(2) = D(8) \quad E(3) = D(7)$

$E(4) = D(6) \quad E(5) = D(5)$

$E(6) = D(4) \quad E(7) = D(3)$

$E(8) = D(2) \quad E(9) = D(1)$

In general, $E(m) = D(10 - m)$ where m will run from 1 to 9, the threshold value. In the same way.

$D(1) = E(9) \quad D(2) = E(8) \quad D(3) = E(7)$

$D(4) = E(6) \quad D(5) = E(5)$

$D(6) = E(4) \quad D(7) = E(3)$

$D(8) = E(2) \quad D(9) = E(1)$

In general, $\mathbf{D(m) = E(10 - m)}$ where m will run from 1 to 9, the threshold value.

Note: $E(1) = D(n^2)$ And $D(1) = E(n^2)$

where n^2 is maximum threshold which is 9 for 3 x 3 structuring element. Here $E(1)$, $D(n^2)$ are pure erosions, and $D(1)$, $E(n^2)$ are pure dilations.

5.2.2. $5/5$ Structuring Element:

If the Tables 3 and 4 are studied carefully, which provide dilated and eroded values, the conclusion may be got that eroded value at one threshold is equal to dilated value at some other threshold. For example E(1) is equivalent to D(25) , because same output value is obtained for having threshold value 1 for soft erosion and threshold value 25 for soft dilation, in $5/5$ environment.

So if we study carefully, the following equalities will be observed.

$E(1) = D(25) \quad E(2) = D(24)$

$E(3) = D(23) \quad E(4) = D(22)$

$E(5) = D(21) \quad E(6) = D(20)$

$E(7) = D(19) \quad E(8) = D(18)$

$E(9) = D(17) \quad E(10) = D(16)$

$E(11) = D(15) \quad E(12) = D(14)$

$E(13) = D(13) \quad E(14) = D(12)$

$E(15) = D(11) \quad E(16) = D(10)$

$E(17) = D(9) \quad E(18) = D(8)$

$E(19) = D(7) \quad E(20) = D(6)$

$E(21) = D(5) \quad E(22) = D(4)$

$E(23) = D(3) \quad E(24) = D(2)$

$E(25) = D(1)$

$E(1)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

$D(25)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

In the same way, E (2) is shown to be equivalent to D (24).

$E(2)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1

$D(24)$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1

In general, $E(m) = D(26 - m)$ where m will run from 1 to 25, the threshold value.

Note: $E(1) = D(n^2)$ And $D(1) = E(n^2)$ where n^2 is maximum threshold which is 25 for $5/5$ structuring element. Here $E(1)$, $D(n^2)$ are pure erosions, and $D(1)$, $E(n^2)$ are pure dilations.

In the same way

$D(m) = E(26 - m)$ Where m will run from 1 to 25, the threshold value. $D(1) = E(n^2)$ and $E(1) = D(n^2)$ where n^2 is maximum threshold which is 25 for $5/5$ structuring element. Here $E(1)$, $D(n^2)$ are pure erosions, and $D(1)$, $E(n^2)$ are pure dilations.

Here $n = 5$ so $n^2 = 25$ so $E(1)$, $D(25)$ are pure erosions and $D(1)$, $E(25)$ are pure dilations.

5.2.3. $7/7$ Structuring Element:

In this situation also we can have the equalities like above because here also dilation at one threshold will be equal to erosion at the other threshold. Here the threshold value will run from 1 to 49.

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$E(1) = D(49)$ $E(2) = D(48)$
$E(3) = D(47)$
$E(4) = D(46)$ $E(5) = D(45)$
$E(6) = D(44)$
……………………………………………………………
…
……………………………………………………………
$E(47) = D(3)$ $E(48) = D(2)$
$E(49) = D(1)$

In general, $E(m) = D(50 - m)$ where $m = 1$ to 49.
Here $E(1)$, $D(49)$ are pure erosions. $D(1)$, $E(49)$ are pure dilations.
In the same way we can have the equalities like
$D(1) = E(49)$
$D(2) = E(48)$
… … … … …
$D(49) = E(1)$
So $D(m) = E(50 - m)$ where $m = 1$ to 49.

5.2.4. $^{9}/_{9}$ Structuring Element:

In this situation also we can have the equalities like above because here also dilation at one threshold will be equal to erosion at the other threshold. Here the threshold value will run from 1 to 81.
$E(1) = D(81)$ $E(2) = D(80)$
$E(3) = D(79)$
$E(4) = D(78)$ $E(5) = D(77)$
$E(5) = D(77)$
……………………………………………………………
…………………………………..
$E(79) = D(3)$ $E(80) = D(2)$
$E(81) = D(1)$

In general, $E(m) = D(82 - m)$ where $m = 1$ to 81.
Here $E(1)$, $D(81)$ are pure erosions. $D(1)$, $E(81)$ are pure dilations.
In the same way we can have the equalities like
$D(1) = E(81)$
$D(2) = E(80)$ $D(3) = E(79)$ … … … … …
$D(80) = E(2)$ $D(81) = E(1)$
So $D(m) = E(82 - m)$ where $m = 1$ to 81.

5.2.5. $^{11}/_{11}$ Structuring Element:

In this situation also we can have the equalities like above because here also dilation at one threshold will be equal to erosion at the other threshold. Here the threshold value will run from 1 to 121.
$E(1) = D(121)$ $E(2) = D(120)$
$E(3) = D(119)$
$E(4) = D(118)$ $E(5) = D(117)$
$E(6) = D(116)$
……………………………………………………………
$E(95) = D(27)$ $E(95) = D(27)$
$E(95) = D(27)$
……………………………………………………………
……
$E(119) = D(3)$ $E(120) = D(2)$
$E(121) = D(1)$

In general, $E(m) = D(122 - m)$ where $m = 1$ to 121.
Here $E(1)$, $D(121)$ are pure erosions. $D(1)$, $E(121)$ are pure dilations. In the same way we can have the equalities like $D(1) = E(121)$ $D(2) = E(120)$
$D(3) = E(119)$
……………………………………
$D(120) = E(2)$ $D(121) = E(1)$
So, $D(m) = E(122 - m)$ where $m = 1$ to 121.
In the same way we can have equalities in soft erosion and soft dilation in various dimensions of structuring elements, $^{13}/_{13}$, $^{15}/_{15}$, $^{17}/_{17}$, $^{19}/_{19}$, $^{21}/_{21}$, $^{23}/_{23}$,……..

5.2.6. General case: …… (6)

For structuring element size: $^{w}/_{w}$

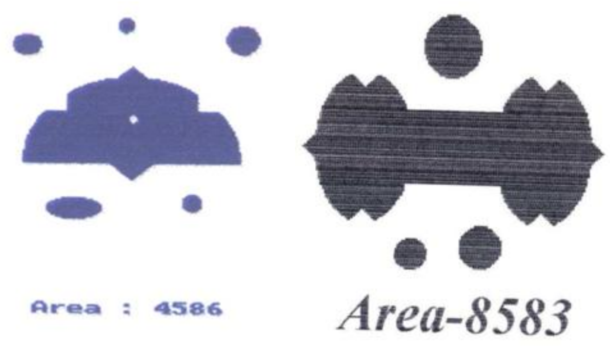
- $E(m) = D(w^2 + 1 - m)$
- $D(m) = E(w^2 + 1 - m)$

By the same logic.

6. RESULTS AND ANALYSIS

In this section the results of experiments are presented. Actually two diagrams are taken, a Semi circle shape and a dumbbell shape. On these images various morphological and soft morphological operations are applied. The output is got in the form of tables, diagrams and graphs, around 1000 pages. But here some important as well as samples outputs are presented.

Images: Semi circle and dumbbell



Semi-circle dumbbell

Fig – 1

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TABLES RELATING SOFT DILATION AND SOFT EROSION IN MULTI SCALE ENVIRONMENT.

In the following output soft erosion at threshold 1is equal to soft dilation at threshold 9,which is equal to 4004 in the case of 3/3 window size. In the same way, soft erosion at threshold 2 is equal to soft dilation at threshold 8, which is equal to 4142 in the same window size which is equal to 3/3. In the same way in the case of 5/5 window size ,soft dilation at threshold 1 is equal to soft erosion at threshold 25. In the same way, soft dilation at threshold 2 is equal to soft erosion at threshold 24 , which is equal to 5706 in the same window size which is equal to 5/5. In this way the results available in the following tables are self explanatory. The tables are available for 7/7,9/9,11/11,13/13 …..(i.e. in multi scale environment) in authors data base. The diagrams are also available in authors data base.(Images with various thresholds of soft erosion and soft dilation in multi scale environment) (6)

Window size 3/3		
Threshold values	Area of soft dilated image	Area of soft eroded image
1	5210	4004
2	5046	4142
3	4939	4215
4	4655	4480
5	4583	4583
6	4480	4655
7	4215	4939
8	4142	5046
9	4004	5210

Window size5/5		
Threshold values	Area of soft dilated image	Area of soft eroded image
1	5070	3469
2	5706	3599
3	5597	3676
23	3676	5597
24	3599	5706
25	3469	5070

In the same way the results of soft erosion and soft dilation for various window sizes are available in authors database in the form of tables, pictures etc.

7. CONCLUSION

In this paper the equivalency property is established in between soft erosion and soft dilation in multi scale environment.

The relevant formulae are for structuring element size: w/w

- $E(m) = D(w^2 + 1 - m)$
- $D(m) = E(w^2 + 1 - m)$

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BIOGRAPHY

Kompella Venkata Ramana has done his B.E (E.C.E) and M.E (COMPUTER ENGINEERING) from ANDHRA UNIVERSITY, VISAKHAPATNAM, INDIA. He has started his carrier as LECTURER in N.I.T. (W). Later he shifted to ANDHRA UNIVERSITY. At present he is working as ASSOCIATE PROFESSOR in the department of computer science &systems engineering in ANDHRA UNIVERSITY. His areas of interest are Image Processing, Formal Languages and Automata Theory, Compiler Design and Systems Programming. He has written books on the above areas. He has experience of more than twenty five years in teaching and guided more than one hundred Theses in M.Tech. Level, majority of them are in Image Processing.